Example 2- FM radio LPDA Design

- Based on work of R. E. Carrel.

1. Select or specify design parameters
   a. Desired directivity (gain) **8 dBi**
   b. Frequency range (f_{high} and f_{low}) $\boldsymbol{f_{low} = 85}$ **MHz,** $\boldsymbol{f_{high} = 110}$ **MHz**
   c. Desired input impedance R_0 (real) $\boldsymbol{R_0 = 75\ \Omega}$

2. Use graph [Balanis 3rd Edn., Figure 11.13, p. 631], which shows contours of constant directivity versus σ (relative spacing) and τ (scale factor), to select σ and τ for the desired directivity.

$\underline{\sigma = 0.159}$ and $\underline{\tau = 0.865}$

3. Calculate the apex half angle α using-

$$\alpha = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right) = \tan^{-1}\left(\frac{1-0.865}{4(0.159)}\right) \quad \Rightarrow \quad \underline{\alpha = 11.984^\circ}$$

$$\text{apex angle} \quad \Rightarrow \quad \underline{2\alpha = 23.968^\circ}$$

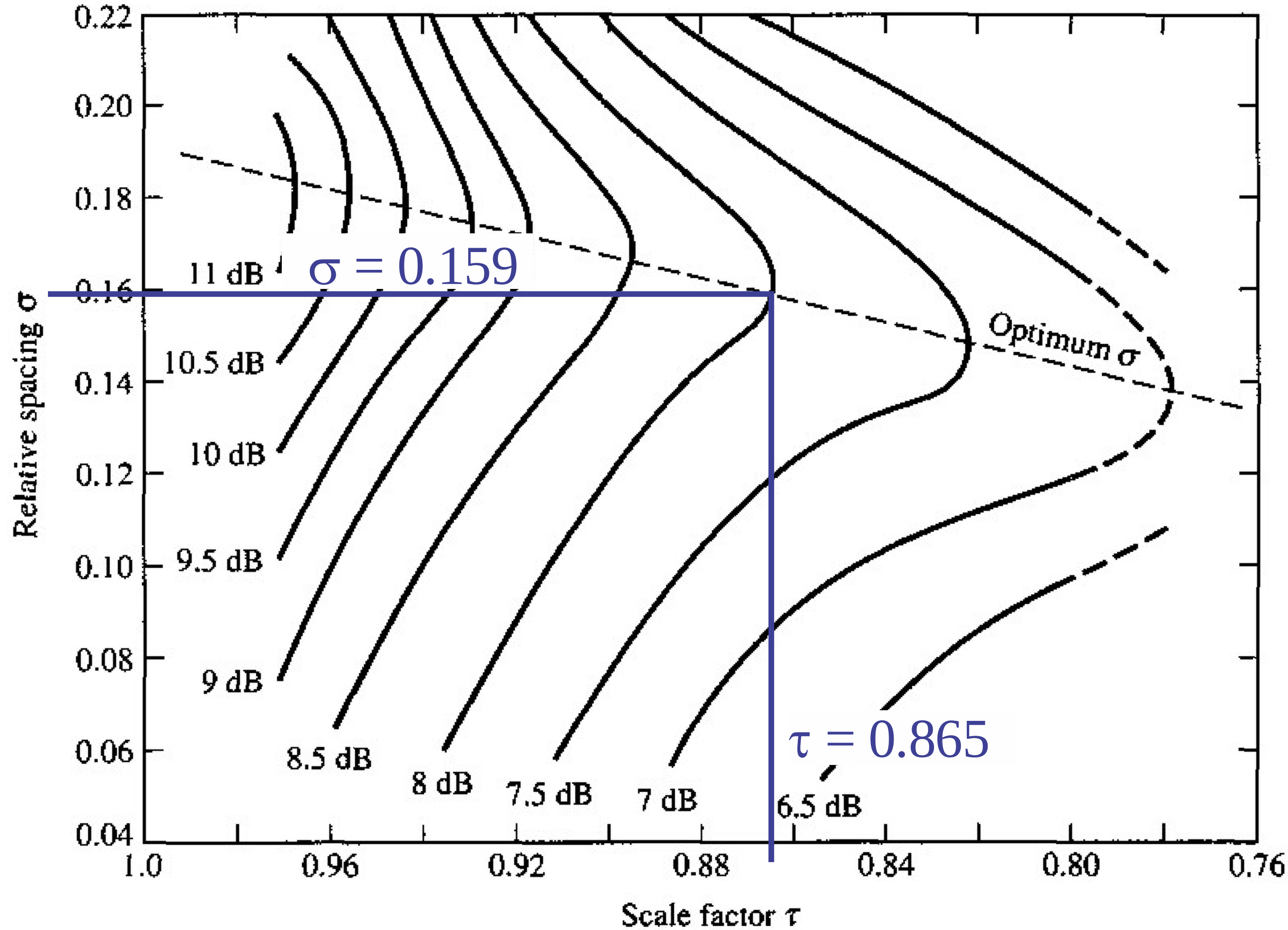


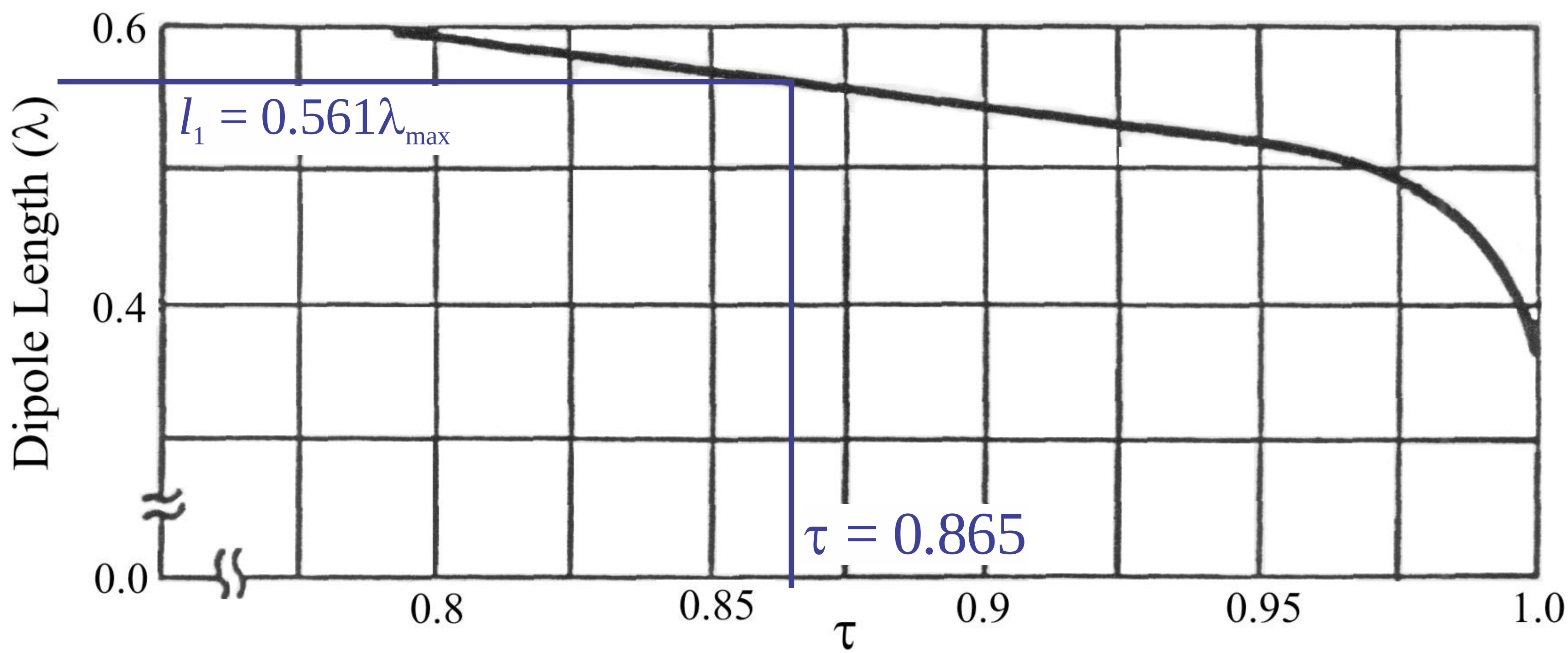
Figure 11.13 Computed contours of constant directivity versus σ and τ for log-periodic dipole arrays. [Balanis 3rd Edn., p. 631]

4. Find length l_1 (Note: start count of elements with longest) of the **longest** element of LPDA

   - take length in wavelengths from graph if using optimum σ and τ;

     $\lambda_{max} = c/f_{low} = 3\times10^8 / 85\times10^6 = 3.5294$ m $\Rightarrow$ **$l_1 = 0.561\,\lambda_{max} = 198$ cm**

   - else, use $\lambda_{max}/2$ where $\lambda_{max} = c / f_{low}$ is the wavelength at the lowest frequency in the desired frequency range. (N/A)

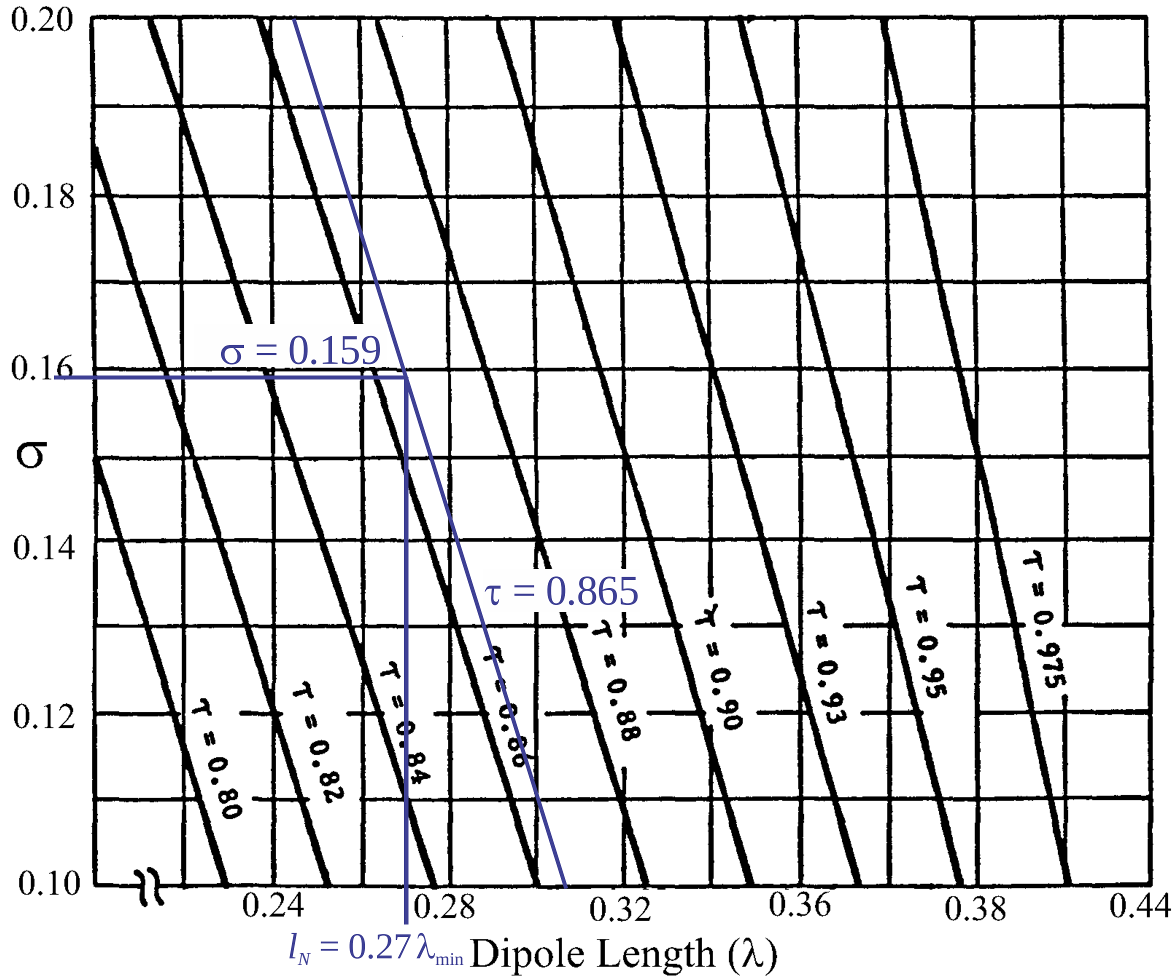


Measured length, normalized by λ_{max}, of longest dipole in LPDA versus optimum σ and τ.

5. Find length l_N of the **shortest** element of the LPDA

- take length in wavelengths from graph where $\lambda_{min} = c / f_{high}$ is the wavelength at the highest frequency in the desired frequency range.

$\lambda_{min} = c/f_{high} = 3\times10^8 / 110\times10^6 = 2.7273$ m $\Rightarrow$ **$l_N = 0.27\,\lambda_{min} = 73.64$ cm**



Estimated length, normalized by λ_{min}, of shortest dipole in LPDA versus σ and τ.

6. Calculate location R_1 of **longest** element (as measured from the apex)-

$$R_1 = \frac{l_1}{2}\cot(\alpha) = \frac{198}{2\tan(11.984^\circ)} \quad \Rightarrow \quad \underline{R_1 = 466.4 \text{ cm}}$$

7. Calculate the total bandwidth B_s, includes additional bandwidth B_{ar} due to active region, using the specified bandwidth B-

$$B = f_{high} / f_{low} = 110 \text{ MHz}/85 \text{ MHz} = \underline{\mathbf{1.29412}}$$

$$B_{ar} = 1.1 + 7.7\,(1-\tau)^2 \cot(\alpha) = 1.1 + 7.7\,(1-0.865)^2 \cot(11.984^\circ) = \underline{\mathbf{1.76112}}$$

$$B_s = B_{ar} \cdot B = 1.76112 \cdot 1.29412 = \underline{\mathbf{2.2791}}$$

8. Calculate the approximate number N of elements required for design

$$N = 1 + \log_{10}(B_s)/\log_{10}(1/\tau) = 1 + \log_{10}(2.2791)/\log_{10}(1/0.865) = 6.68 \Rightarrow \underline{\mathbf{N \approx 7}}$$

9. Calculate the approximate distance L_T between the longest and shortest elements.

$$L_T = \frac{l_1}{2}(1 - 1/B_s)\cot(\alpha) = \frac{198}{2}\left(1 - \frac{1}{2.2791}\right)\cot(11.984^\circ) \Rightarrow \underline{L_T = 261.76 \text{ cm}}$$

10. Calculate the location R_2 (from the apex) and length l_2 of the second longest element using the scale factor τ, R_1, and l_1 -

$$R_2 = R_1\,\tau = 466.4(0.865) \quad \Rightarrow \quad \underline{\mathbf{R_2 = 403.44\ cm}}$$

and

$$l_2 = l_1\,\tau = 198\,(0.865) \quad \Rightarrow \quad \underline{\mathbf{l_2 = 171.27\ cm}}$$

11. Recursively calculate the location R_{n+1} and length l_{n+1} of the n+1th element(s) using the scale factor τ, R_n, and l_n –

e.g., $R_{n+1} = R_n\,\tau = R_n\,(0.865)$ & $l_{n+1} = l_n\,\tau = l_n\,(0.865)$.

Stop when l_{n+1} is less than or equal to l_N (calculated in step 5.).

12. Count actual number of elements and calculate actual length of LPDA (compare to approximate calculations in steps 8. & 9.).

- Steps 10-12 done using MS-Excel spreadsheet shown on next page.

Steps 10. & 11. **Calculate:** $R_{n+1} = R_n * \tau$ **and** $l_{n+1} = l_n * \tau$

where $\tau =$ **0.865**

n	***ln* (cm)**	***Rn* (cm)**
1	198	466.4
2	171.270	403.436
3	148.149	348.972
4	128.148	301.861
5	110.848	261.110
6	95.884	225.860
7	82.940	195.369
8	71.743	168.994

Stop since $l_8 < l_N = 73.64$ cm

Step 12. Actual # of elements & antenna length

***N*approx = 7** **LT = 261.76 cm**

***N*actual = 8** ***L*actual = *R*1 - *R*8 = 297.406 cm**

13. Select a length to diameter ratio $K = l/d$ for the elements of the LPDA. This choice is a compromise between mechanical strength for the largest and smallest elements, available tubing sizes, and the selected diameter of the boom.

Choose boom diameter ***D* = 7/8" = 2.223 cm**

If $d_1 = 5/8" = 1.5875$ cm, then $K_1 = l_1/d_1 = 198/1.5875 = 124.7 \Rightarrow$ ***K* = 125**.

Check if this works on smallest element-

$d_8 = l_8/K = 71.7/125 = 0.574$ cm (OK)

14. Calculate the diameter $d_n = l_n / K$ for each element. Then, select the closest available tube/pipe/rod diameter to the calculated value.
15. Calculate the actual length to diameter ratio K_n for each element and the average length to diameter ratio K_{ave} after quantization. Check for unusually large deviations from desired K (may want to go back to step 13. and select another value of K).
    - Steps 14-15 done using MS-Excel spreadsheet shown below.

Step 13. Select K = 125

Step 14. & 15. $d_n = l_n / K$ $K_{actual} = l_n / d_{quantized}$

		Exact	Quantized		Actual	ΔK=
n	l_n (cm)	d_n (cm)	d_n (cm)	d_n (in)	K_n	K_n-K
1	198	1.584	1.5875	5/8	124.724	-0.28
2	171.270	1.370	1.4290	9/16	119.853	-5.15
3	148.149	1.185	1.1110	7/16	133.347	8.35
4	128.148	1.025	1.0320	13/32	124.175	-0.83
5	110.848	0.887	0.8730	11/32	126.974	1.97
6	95.884	0.767	0.7940	5/16	120.761	-4.24
7	82.940	0.664	0.6350	1/4	130.614	5.61
8	71.743	0.574	0.5560	7/32	129.034	4.03
				Average K =	**126.185**	

16. Calculate the approximate average characteristic impedance of the active region elements-

$Z_a = 60 \ln(2 X K_{ave} / \pi) = 60 \ln[2(0.58996)126.185/\pi] \Rightarrow$ $\underline{\mathbf{Z_a = 231.508\ \Omega}}$

where $X = 8 \tau \sigma / (1 + \tau) = 8(0.865)0.159/(1+0.865) = 0.58996.$

17. Find the characteristic impedance of the unloaded transmission line Z_0 for the desired input impedance R_0-

$$Z_0 = \frac{R_0^{\ 2}}{4Z_a X} + R_0 \sqrt{\left(\frac{R_0}{4Z_a X}\right)^2 + 1} = \frac{75^2}{4(231.5)0.59} + 75\sqrt{\left(\frac{75}{4(231.5)0.59}\right)^2 + 1}$$

$$= 10.296 + 75.7035 = \underline{86.000\,\Omega}$$

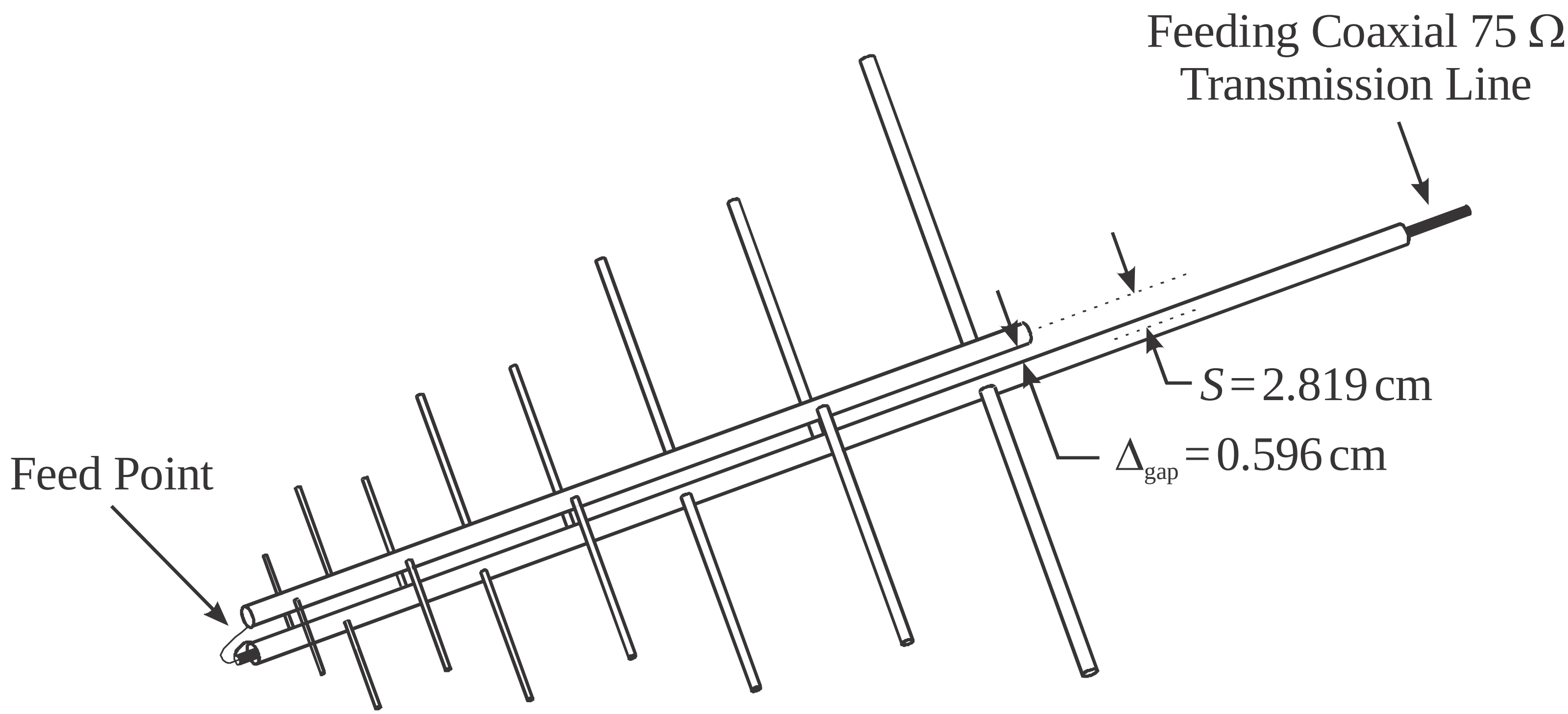
18. Calculate the **center-to-center** spacing S of the booms using the unloaded, cylindrical, twin-lead transmission line formula-

$S = D \cosh(Z_0/120) = 2.223 \cosh(86/120) \Rightarrow$ $\underline{\mathbf{S = 2.819\ cm}}$

where D is the diameter of the booms (assumed to be identical). The air gap Δ_{gap} between the inner surfaces of the booms is-

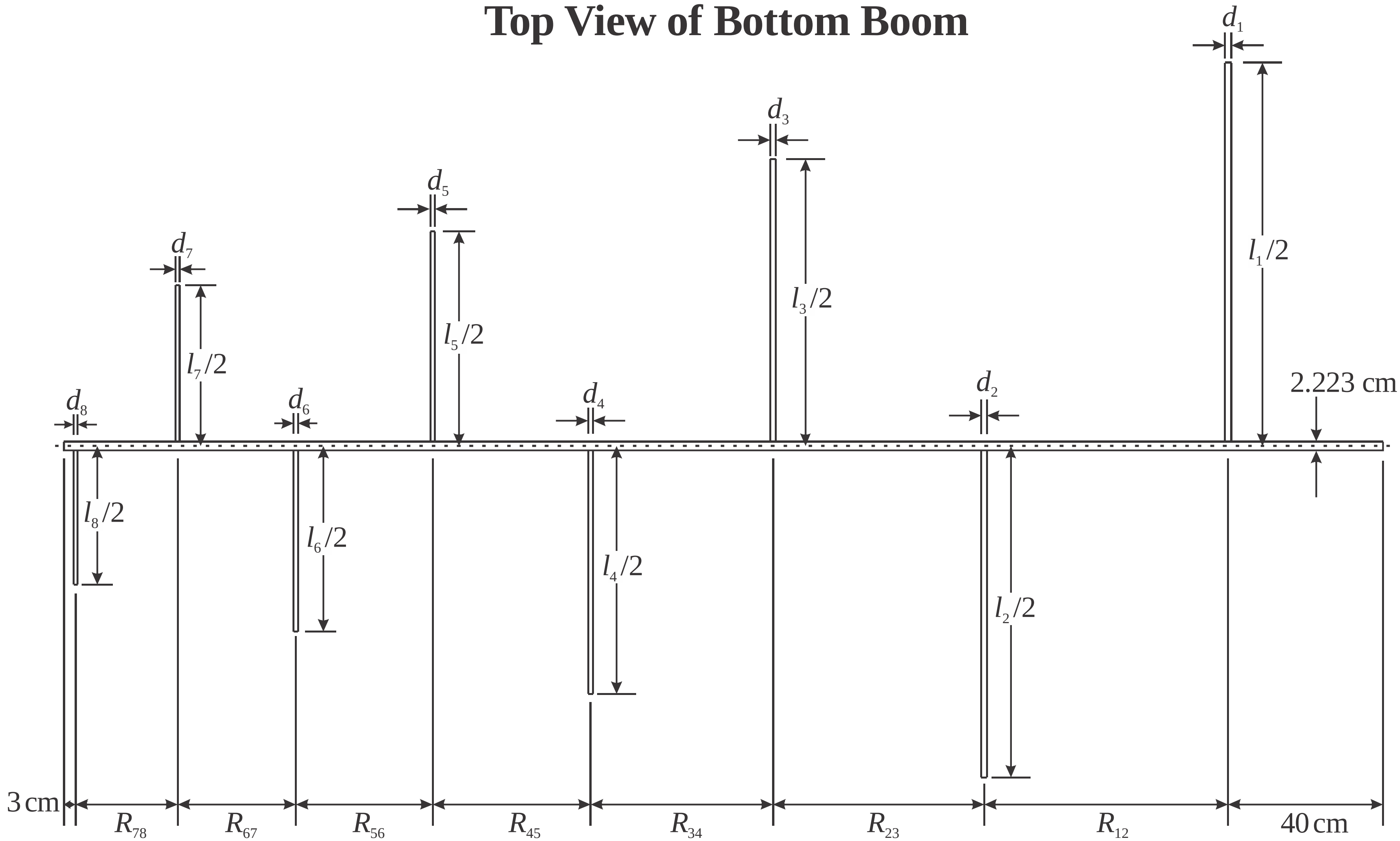
$\Delta_{gap} = S - D = 2.819 - 2.223 \Rightarrow$ $\underline{\mathbf{\Delta_{gap} = 0.596\ cm = 5.96\ mm}}$.

Perspective View of 8 Element LPDA for FM radio



Not to scale. Dimensions shown on following views of bottom and top booms

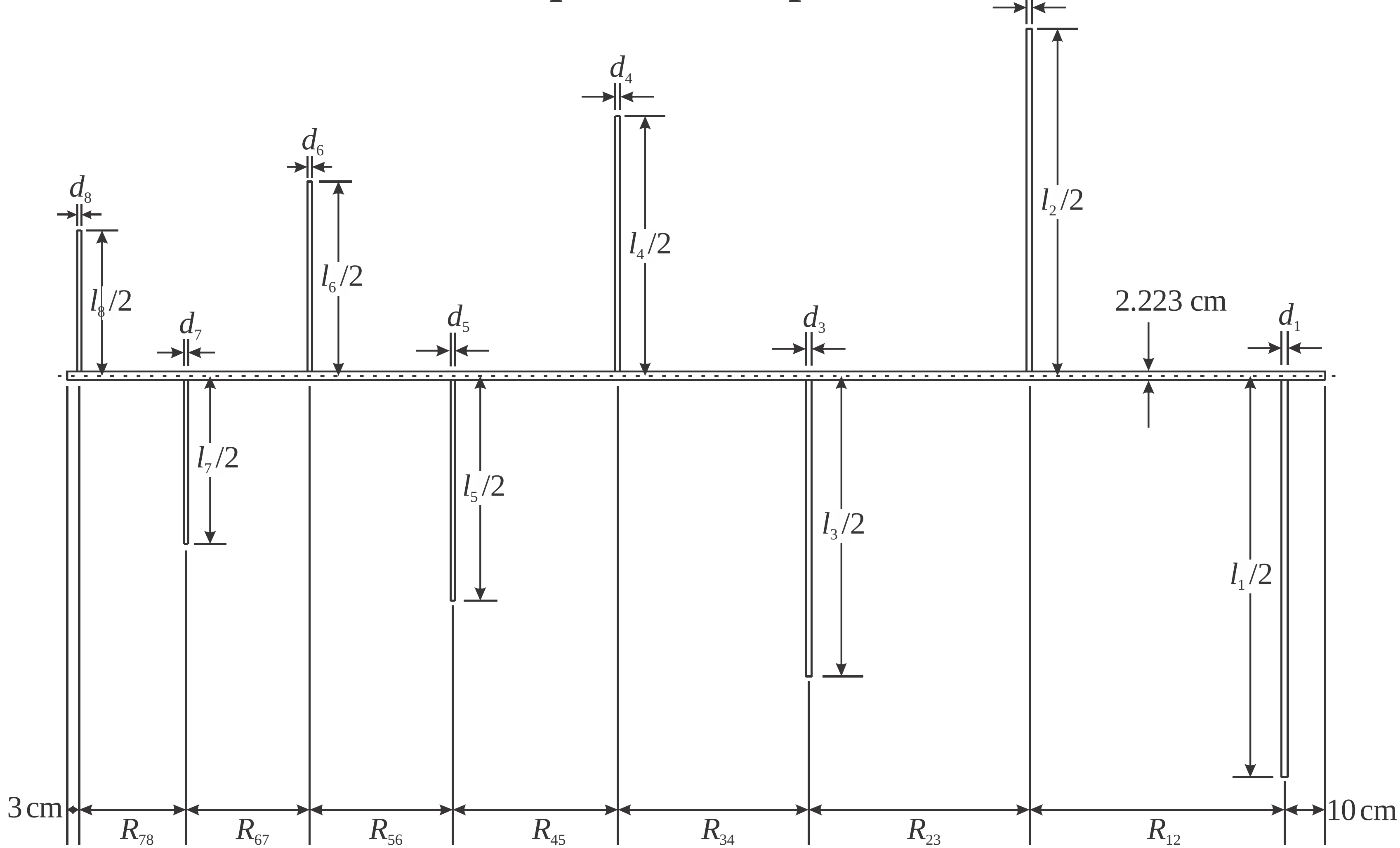
Top View of Bottom Boom



n	ln (cm)	dn (cm)	Rn (cm)	element-element spacing (cm)	
1	198	1.5875	466.4		
2	171.270	1.4290	403.436	R1 - R2	62.964
3	148.149	1.4290	348.972	R2 - R3	54.464
4	128.148	1.2700	301.861	R3 - R4	47.111
5	110.848	1.1110	261.110	R4 - R5	40.751
6	95.884	1.1110	225.860	R5 - R6	35.250
7	82.940	1.0320	195.369	R6 - R7	30.491
8	71.743	0.9525	168.994	R7 - R8	26.375

scale 0.03” = 1 cm

Top View of Top Boom



n	ln (cm)	dn (cm)	Rn (cm)	element-element spacing (cm)	
1	198	1.5875	466.4		
2	171.270	1.4290	403.436	R1 - R2	62.964
3	148.149	1.4290	348.972	R2 - R3	54.464
4	128.148	1.2700	301.861	R3 - R4	47.111
5	110.848	1.1110	261.110	R4 - R5	40.751
6	95.884	1.1110	225.860	R5 - R6	35.250
7	82.940	1.0320	195.369	R6 - R7	30.491
8	71.743	0.9525	168.994	R7 - R8	26.375

scale 0.03” = 1 cm